Physics Challenge Lincoln 2020-2021

For a chance to win the first prize (£100 Amazon voucher), submit your typed or neatly written (**and scanned in a single pdf file**) solutions of the following problems to **physics@lincoln.ac.uk** with subject "Physics Challenge 2020". Please include your full name, postal address and email, as well as the name and address of your school. **The closing date is 31 January, 2021.** The prize-giving ceremony will be organized in the form of an online meeting by the end of January 2021. It is possible to win a prize even if you have not completed all of the questions, so you are encouraged to submit solutions if you do only some of the problems. The competition is open to all young pre-university people in UK aged 15–18 years. It is not open to current university students. See full Terms and Conditions.

Note: all answers must be thoroughly justified.

Q1:
When we use a plumb bob as a vertical reference line in construction work, we assume that it points towards the centre of the Earth. However, all objects on Earth are subject to the gravitational influence of the Moon as well as the Earth. It therefore appears reasonable to assume that the plumb bob actually points towards the centre of mass of the Earth-Moon system. If this were the case, the plumb bob would be directed towards a point about 4800km away from the centre of the Earth (dependent on the phase of the Moon). In practice however, this doesn't seem to be the case. Explain why.

Q2:
The Lorentz force acting on a point charge q moving with velocity $\vec{v}$ subject to a magnetic field $\vec{B}$ is $\vec{F}_{\text{Lorentz}} = q\vec{v} \times \vec{B}$.

(a) Show that this force cannot do any work on the point charge.

The magnetic force per unit length exerted between two vertical wires each carrying the same current I is attractive, with magnitude

$$\left|\vec{F}_{\text{wires}}\right| = \frac{KI^2}{L}$$

where K is a positive constant and L is the separation between the wire.

(b) Show that, upon changing the separation L, the work done by this magnetic force on one of the wires is non-zero.

(c) Explain how a magnetic force can do zero work on a moving point-charge but still do work on a current-carrying-wire.

Q3:

Real fluids tend to have a density-dependent pressure which deviates from the simple ideal gas law. Finding corrections to the ideal gas law and more accurate equations for the pressure as a function of density is crucial for both industry and fundamental research. One theoretical tool used to do this is the so-called virial expansion, which expresses the pressure P of a fluid as a function of its particle density ρ as follows

$$\frac{P}{\rho RT} = 1 + \sum_{i=1}^{+\infty} B_i \, \eta^i$$

where R is the ideal gas constant, T is the absolute temperature expressed in Kelvin, $\eta = v_{\mathrm{part}} \, \rho$ is the packing fraction of the fluid (v_{part} is the effective volume of a particle) and the B_i coefficients are real constants. For a fluid comprising only hard spheres, the first three B_i coefficients have been evaluated to be $B_1 = 4, B_2 = 10$ and $B_3 = 18.365$. However, the time it takes to evaluate a given B_i roughly increases exponentially with i. An approximate tractable approach consists then in noticing that, when rounded to the nearest integer, the B_i coefficients appear to closely follow the relation $B_i = a_1 i^2 + a_2 i + a_3$.

Find the values of the coefficients a_1, a_2 and a_3 given the three B_i values provided above and derive an approximate closed-form expression (i.e. not as a series) for the pressure P of a hard-sphere fluid.

Q4:

Consider a 3cm long Tungsten filament in a vacuum bulb. Estimate what the cross-sectional area A of the filament should be if we want it to emit light centred around $\lambda = 850\mathrm{nm}$ when subjected to a voltage of 220V.