Physics Challenge Lincoln 2021-2022

For a chance to win the first prize (£100 Amazon voucher), submit your typed or neatly written (**and scanned in a single pdf file**) solutions of the following problems to **physics@lincoln.ac.uk** with subject "Physics Challenge 2021". Please include your full name, postal address and email, as well as the name and address of your school. **The closing date is 6 January, 2022.** The prize-giving ceremony will be organized in the form of an in-person or online meeting (depending on governmental guidance) by the end of February 2022. It is possible to win a prize even if you have not completed all of the questions, so you are encouraged to submit solutions even if you do only some of the problems. The competition is open to all young pre-university people in UK aged 15–18 years. It is not open to current university students. See full Terms and Conditions.

Note: all answers must be thoroughly justified.

Q1:

A bob of mass m is hanging from a massless string of length ℓ whose other extremity is fixed at a point O located at height $h > \ell$. The bob is initially at rest in an inertial frame and is given an initial horizontal velocity of magnitude v_0 in that frame. The deviation of the bob from its initial downward vertical position is characterised by an angle θ. We shall denote g the magnitude of the downward (vertical) acceleration of the weight on Earth.

a) Using Newtonian mechanics show that the tension in the string will vanish at a certain angle θ^* along the trajectory of the bob if the initial speed v_0 lies in the range $[\sqrt{2g\ell}, \sqrt{5g\ell}]$. Explain qualitatively what happens to the bob's trajectory in such cases (you may use a diagram).

b) If v_0 belongs to the range mentioned in Q1a), plot the graph of the speed $v^*(v_0)$ of the bob at the point of the trajectory where the tension in the string vanishes and identify the range of values taken by $v^*(v_0)$ on that interval.

Q2:

Consider two spherical bodies 1 and 2 of mass and radius (m_1, R_1) and (m_2, R_2) respectively. The two bodies lie at positions $\vec{r}_1$ and $\vec{r}_2$ as seen from an inertial frame. The two masses solely interact via Newton's Universal law of Gravitation:

$$\vec{F}_{12} = -\vec{F}_{21} = -\frac{Gm_1m_2(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3},$$

where $\vec{F}_{12}$ (resp. $\vec{F}_{21}$) is the gravitational force of mass m_1 (resp. m_2) onto m_2 (resp. m_1), G is the universal constant of gravitation and $|\vec{r}_2 - \vec{r}_1|$ is the magnitude of the vector $\vec{r}_2 - \vec{r}_1$ joining m_1 to m_2.

a) Assuming that the centres of the two bodies are initially at rest and separated by a distance d, show that the collision time T_{coll} for them to enter into contact from their sole mutual gravitational interaction reads:

$$T_{\text{coll}} = \frac{d^{3/2}}{\sqrt{G(m_1 + m_2)}} f\left(\frac{d}{R_1 + R_2}\right),$$

where $f(x)$ is a factor of order $O(1)$ whose expression must be specified (you may use an integral form). Provide an approximation of T_{coll} when $d \gg (R_1 + R_2)$.

b) Consider now the case of two identical-size test masses $m_A = 6\times10^9\text{kg}$ and $m_B = 10^3\text{kg}$ both initially at rest (in an approximate inertial frame) at a distance $h = 3000\text{km}$ from the surface of the Moon and sufficiently far apart for their mutual interactions to be negligible relative to their respective interaction with the Moon. Consequently, each test mass is going to fall onto the Moon according to the law derived in Q2a). Find an estimate of the time difference between the collision times of the two objects. Would this time difference be measurable with the currently available technology?

Q3

16^{th} century astronomy was mostly done with the naked eye. Given that the background of the fixed stars was not moving over a yearly period, Copernicans had to assume that these stars were so far away that their relative motion with respect to the Earth was imperceptible. This question pertains to inferred estimates of the size of these fixed stars.

a) Sirius, the brightest star in the night sky, is approximately 1 million times further away from us than Venus. Assuming that its image is the same size as that of Venus on our retina, how large do we expect Sirius to be if we assume the cornea of the eye to be a converging thin lens and the distance from the cornea to the image formed on the retina to be fixed? The radius of Venus is about 6000km.

b) Given that on average Pluto is at a distance of about 6×10^9 km from the Sun, does your result found in Q3a) seem reasonable for the size of a star? Discuss your answer by using the concept of angular resolution of an optical system.

Q4

According to classical mechanics, the time-dependent position $x(t)$ of a mass m solely subject to the force exerted by a spring of strength k is $x(t) = x_0 \cos(\omega t)$, where $\omega = \sqrt{k/m}$ is the angular frequency of oscillation and x_0 a real number.

a) Show that given the solution above we have

$$\frac{\Delta p^2}{2m} = \frac{1}{2}k\Delta x^2 = \frac{E}{2}\,, \tag{4.1}$$

where $p(t) = m\frac{dx}{dt}$, E is the energy of the system and where for any function of time $f(t)$, $\Delta f^2 \equiv \langle (f(t) - \langle f \rangle)^2 \rangle$ with

$$\langle f \rangle \equiv \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} f(t)dt.$$

b) A hallmark of quantum mechanics is the so-called Heisenberg inequality stating that $\Delta x \Delta p \geq \hbar/2$, where $\hbar \equiv h/2\pi$ is the reduced Planck's constant. This inequality prevents the energy E of the system to ever reach zero. In fact, the minimum energy E_0 of a quantum harmonic oscillator is achieved when saturation of the Heisenberg inequality occurs i.e. when $\Delta x \Delta p = \hbar/2$.
Assuming Eq. (4.1) still holds in quantum mechanics, find the expression of the minimum energy of a harmonic oscillator in quantum mechanics.