Physics Challenge Lincoln 2022-2023

For a chance to win the first prize (£100 Amazon voucher), submit your typed or neatly written (**and scanned in a single pdf file**) solutions of the following problems to **physics@lincoln.ac.uk** with subject "Physics Challenge 2022". Please include your full name, postal address and email, as well as the name and address of your school. **The closing date is 6 January, 2023.** The prize-giving ceremony will be organized in the form of an in-person meeting by the end of February 2023. It is possible to win a prize even if you have not completed all of the questions, so you are encouraged to submit solutions even if you do only some of the problems. The competition is open to all young pre-university people in UK aged 15–18 years. It is not open to current university students. See full Terms and Conditions.

Note: all answers must be thoroughly justified.

Q1:
In a suitable inertial frame, a mass m is free to move along a massless hoop of radius R which is rotating about its vertical axis of symmetry with an angular velocity ω (cf. Figure 1 below). The plane containing the hoop is always perpendicular to the surface the hoop is spinning on. The same part of the hoop remains in contact with the surface throughout the course of the motion (the hoop is not rolling). Determine the expression of the stationary angle γ_s at which the mass settles as a function of ω, R, m and g the downward acceleration on Earth.

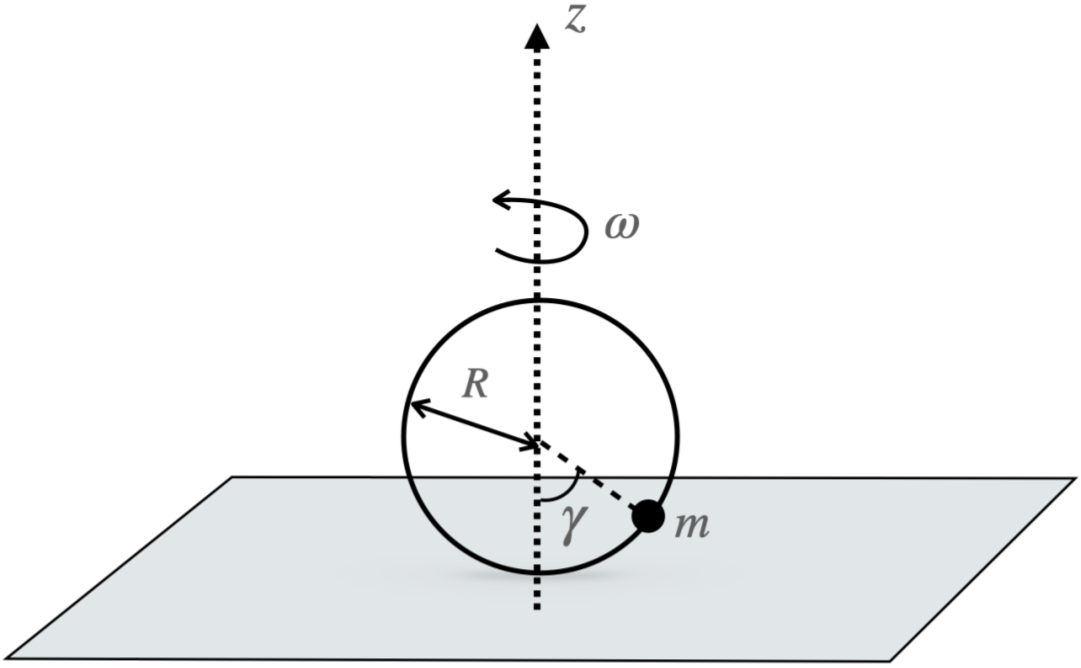

Figure 1: A mass freely moving on a spinning massless hoop.

Q2:
An exact stationary and spherically symmetric metric in an empty region of space-time around a mass M was originally derived by Schwarzschild in 1916[1] and read

$$ds^2 = \left(1 - \frac{R_s}{R}\right) c^2 dt^2 - \frac{dR^2}{\left(1 - \frac{R_s}{R}\right)} - R^2(d\theta^2 + \sin^2\theta\, d\varphi^2),$$

where ds^2 is the square line element in space-time, $R_s \equiv 2GM/c^2$ is the so-called Schwarzschild radius, θ and φ are angles characterising the position of a point on a sphere (θ being the colatitude and φ the longitude), t is a time-coordinate associated to a local clock and $R = (r^3 + R_s^3)^{1/3}$ is an auxiliary radial-like coordinate and r is the true spatial radial spherical coordinate away from the centre of the mass.

(a) Show that when $r \gg R_s$ Schwarzschild's original solution becomes identical to what is currently referred to as 'Schwarzschild's metric' or 'Black-Hole metric' where R is basically substituted by r in the above expression [2]. Which power of R_s/r would be involved in the first order correction to this 'Black-Hole metric'?

(b) We now consider two observers A and B located respectively at r_A and r_B with $r_B < r_A$ and $\theta_A = \theta_B = 0$ and $\varphi_A = \varphi_B$. Observer A sends two light signals separated by a time T_A as read by their local clock. We denote T_B the time interval as measured by the local clock of observer B between the reception of the two signals.
Use the fact that ds^2 is an invariant to determine a relationship between T_A, T_B, r_A and r_B. You may compare expressions obtained depending on whether the metric used is the original or the 'Black-Hole' Schwarzschild metric.

Q3:

(a) Show that within an inertial frame, Newton's 3rd law of motion of action-reaction implies the conservation of the total linear momentum for any isolated systems of N point masses mutually interacting with each other.

(b) Consider now an isolated system of two point charges q_1 and q_2 located at $\vec{r}_1$ and $\vec{r}_2$ respectively and moving at velocities $\vec{v}_1$ and $\vec{v}_2$ respectively.
Assuming that they interact via the Lorentz force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$, where $\vec{E}$ and $\vec{B}$ are electric and magnetic fields acting on a test charge q moving at velocity $\vec{v}$, determine $\vec{F}_{12}$ (resp. $\vec{F}_{21}$) the force from charge q_1 onto charge q_2 (resp. the force from q_2 onto q_1) and discuss their relationship with Newton's 3rd law of action-reaction.

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[1] https://obliviousphysicist.wordpress.com/2022/02/10/would-schwarzschild-have-predicted-black-holes/
[2] https://en.wikipedia.org/wiki/Schwarzschild_metric

Q4:

(a) Consider a spherical droplet of a minor phase A into a dominant phase B [3] with a fixed total volume V of the entire system and fixed total number of particles in the system and imposed temperature.
The pressure in the droplet of phase A is denoted P_A and within phase B the pressure is denoted P_B.
Assuming that the interface between the phases A and B contributes positively to the energy of the system by an amount σ per unit area, determine the relationship between P_A, P_B, σ and R_{eq} the radius of the droplet at thermodynamic equilibrium. You may use the fact that the bulk pressure P may be defined as $-\partial F/\partial V$ where F is the bulk (free) energy of a given system.

(b) We now consider two spherical soap bubbles connected by a thin straw. Given that one bubble is smaller than the other, describe with as many details as you can the fate of this system in light of your answer in Q4(a).

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[3] By 'phase' we mean any substance or phase of matter which can form a droplet into another substance or phase of matter while keeping the amount of matter the same.